Multi-objective Reactive Power Compensation with Voltage Security

Rodrigo Ramos, José Vallejos, *Member IEEE* and Benjamín Barán, *Member IEEE*

***Abstract* — This paper presents a new approach using a variant of the Strength Pareto Evolutionary Algorithm (SPEA) to address reactive power compensation in electric systems as a multi-objective problem, turning most traditional constraints into new objective functions.**

The method independently optimizes several figures related with the operation of a Power System such as, amount of investment in reactive power devices, transmission losses and, as an addition to previous approaches, voltage security is included. At the same time, constrains limit other parameters as reliability and voltage profile.

That way, a wide set of optimal solutions, known as Pareto set, is found before deciding which solution best combines different features.

A set of compensation schemes elaborated by specialists using traditional engineering tools is compared to a Pareto set found with the proposed approach using appropriate test suite metrics. Comparison results emphasize outstanding advantages of the proposed computational approach, such as: ease of calculation, better-defined Pareto front and a larger number of Pareto solutions.

***Index Terms* — Reactive power compensation, voltage security, multi-objective optimization, evolutionary algorithms.**

I. INTRODUCTION

REACTIVE power compensation is commonly addressed as a constrained single-objective optimization problem [1-3]. Traditionally, it basically consists in determining an adequate location and size of shunt and/or series capacitor and reactor banks. In this context, the objective function is a linear combination of several factors, such as: investment in reactive power devices, transmission losses and voltage security [4]. Traditional single-objective optimization algorithms usually provide a unique optimal solution. On the contrary, *Multi-objective Optimization Evolutionary Algorithms* (MOEA) independently and simultaneously optimize several parameters turning most traditional constraints into new objective functions. This seems more natural for real world problems where choosing a threshold may seem arbitrary [5]. As a result, a wide set of optimal solutions (Pareto set) may be found. Therefore, an engineer may have a whole set of optimal alternatives before deciding which solution is the best compromise of different (and sometimes contradictory) features.

In the literature, there can be found several papers that address the reactive power compensation as a Multi-objective problem, [6,7], yet most of them consider a linear combination of objectives, thus leading to special cases of optimal power flow.

This paper presents the reactive power compensation problem as a multi-objective problem, where several objective functions are optimized independently. The objective functions selected comprise the following aspects of the problem:

- Investment in compensation devices, such as shunt capacitor banks. This item has both economic and technical importance, since an overcompensated power system can lead to undesired over voltages and oscillations [4]
- Active power losses, which is another key economic aspect in an efficient operation of a power system.
- Voltage security, in order to avoid operation points that could lead to unstable behavior of the system.

To solve the reactive power compensation problem as exposed, this paper presents a new approach based on a modified *Strength Pareto Evolutionary Algorithm* (SPEA) [8, 9, 10], which is a MOEA with an external population of Pareto Optimal solutions that best conform a Pareto Front, provided by a clustering process that saves the most representative solutions. In this paper, a new important objective has been included, regarding the voltage security of the power system.

II. MATHEMATICAL FORMULATION

For the purposes of this paper, the following assumptions where considered in the formulation of the problem:

- only shunt capacitor/reactor banks were considered as reactive power sources,
- shunt-capacitor/reactor bank cost per MVAr is the same for all busbars of the power system;
- power system is considered only at peak load.

Based on these considerations and the previous mentioned in the introduction, tree objective functions F_i (to be optimized) have been identified [4, 9, 11]: F_1 and F_2 are related to investment and transmission losses respectively, while F_5 function is related with the voltage security of the system. Each one of the functions is formulated in what follows:

This work was supported by the Faculty of Engineering (FIUNA) and the National Computing Center (CNC) of the National University of Asuncion (UNA) – Paraguay.

F_1: *Investment in reactive compensation devices*

$$F_1 = \sum_{i=1}^{n} k|B_i| \tag{1}$$

$$k = \begin{cases} \alpha & \text{if } \ 0 \le B_i < B_m \\ \beta & \text{if } \ -B_m < B_i < 0 \end{cases}$$

s.t.: $F_1 \le F_{1m}$;

where: F_1 is the total required investment; F_{1m} is the maximum amount available for investment; B_i is the compensation at busbar i measured in MVAr; B_m is the absolute value of the maximum amount of compensation in MVAr allowed at a single busbar of the system; α is the cost per MVAr of a capacitor bank; β is the cost per MVAr of a reactor bank and n is the number of busbars in the electric power system.

F_2: *Active power losses*

$$F_2 = P_g - P_l \ge 0 \tag{2}$$

Where: F_2 is the total transmission active losses of the power system in MW; P_g is the total active power generated in MW and P_l is the total load of the system in MW.

F_3: *Voltage security*

$$F_3 = \lambda_* \tag{3}$$

where λ_* is a loading factor associated to a critical (unstable) point [12, 13], i.e. a point where the Jacobian of the system becomes singular. This magnitude is calculated using the Continuation Method [14, 15], assuming a proportional load increment through all busbars of the network. Thus, in order to obtain a reasonable stability margin, this parameter has to be maximized.

In summary, the optimization problem to be solved is the following:

$$\text{optimize } \ \mathbf{F} = [F_1 \quad F_2 \quad F_3] \tag{4}$$

where

$$\mathbf{F} = \left[\sum_{i=1}^{n} B_i \quad P_g - P_l \quad \lambda_* \right]$$

is known as *objective vector*, subject to $F_1 \le F_{1m}$ and a set of nonlinear power flow equations [6]:

$$0 = H(x, \rho, \lambda) \tag{5}$$

where $x \in \Re^{ND}$ is the set of dependent variables of the system, while $\rho \in \Re^{NI}$ is the set of independent variables. λ is the loading factor associated to the operating point, referred to a "base" load power level.

To represent the amount of reactive compensation to be allocated at each busbar i, an unknown vector **B**, known as *decision vector* [8], is used to indicate the size of each reactive bank in the power system, i.e.:

$$\mathbf{B} = [B_1 \quad B_2 \quad \cdots \quad B_n], B_i \in \Re, |B_i| \le B_m \tag{6}$$

A basic example of this representation is illustrated in Figure 1, applied to a 6-busbar example.

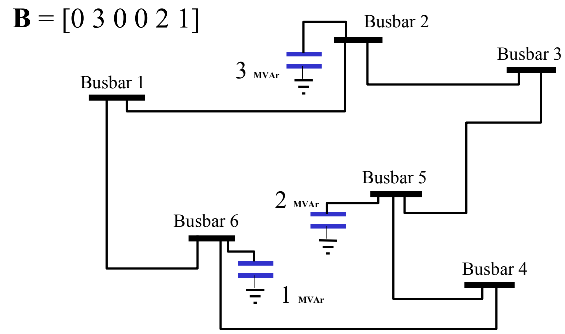

Fig. 1. Example of a decision vector

The set of solutions of a multi-objective optimization problem consists of all decision vectors **B** for which the corresponding objective vectors **F** cannot be improved in any dimension without degradation in another. This set of decision vectors is known as *Pareto Optimal*, represented as P. The corresponding set of objective vectors **F** calculated using equations (1) to (3) conform a set known as *Optimal Pareto Front*, denoted PF [8].

Because the *true* Pareto Optimal Set (termed T_{true}), with its corresponding PF_{true}, are not completely known in practice without extensive calculation (computationally not feasible in most situations), it would be normally enough for practical purposes to find a *known* Pareto Optimal Set, termed P_{known}, with its corresponding Pareto Front PF_{known}, close enough to the true optimal solution [5].

III. PROPOSED METHOD

A new approach based on the Strength Pareto Evolutionary Algorithm was developed for this work. This method, closely related to Genetic Algorithms [16], is based on generating a stored *External Population* composed by the best-known individuals **B** of a general evolutionary population. This external group of solutions conforms P_{known}, available at each moment of the computation, i.e., the best-known approximation to P_{true}. The original SPEA evaluates an individual's fitness depending on the number of decision vectors it dominates in an evolutionary population, i.e., decision vectors that are not better in any objective function F_i, but with a worse objective function F_i for at least one value of i.

SPEA preserves population diversity using Pareto dominance relationship and incorporating a clustering procedure in order to reduce the nondominated set without destroying its characteristics. In general, cluster analysis partitions a collection of m elements into g groups of relatively homogeneous elements, where $g < m$, selecting a representative individual for each of the g clusters. That way, a fixed number of g individuals may be maintained in the external population preserving the main characteristics of the Pareto Front [8].

An important issue with SPEA is its convergence property, assured by *Theorem 4* proved in [5], a characteristic not always present in other MOEAs. Consequently, the algorithm implemented for this work is based on the original SPEA [8], but differs from it in the following aspects:

- *Heuristic Initialization.* A special heuristic method is used to generate the initial population in order to obtain individuals electrically well compensated. The proposed heuristic is based on encouraging compensation at busbars with large number of branches and voltage profile far from the desired value. This is done by using a method summarized as follows:
  a. Choose a total amount of compensation B_{tot}.
  b. For each busbar i of the system, calculate a factor K_i using the following expression:
  $$K_i = \begin{cases} \left(V_i - V_i^*\right) l_i & \text{if } V_i < V_i^* \\ 0 & \text{if } V_i \geq V_i^* \end{cases} \tag{6}$$
  where l_i is the number of branches connected to node i. $K_i = 0$ indicates that no reactive compensation is heuristically assigned to busbar i.
  c. Normalize K_i using:
  $$K_i' = \frac{K_i}{\sum_{i=1}^{n} K_i} \tag{7}$$
  d. Compensate each busbar i with B_i calculated as follows:
  $$B_i = K_i' B_{tot} \tag{8}$$
- *Local Optimization.* A special heuristic technique is implemented to improve individuals based on determining an adequate search direction using the power flow mismatch expression [4]:
  $$\begin{bmatrix} \mathbf{\Delta P} \\ \mathbf{\Delta Q} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 \mathbf{J}_2 \\ \mathbf{J}_3 \mathbf{J}_4 \end{bmatrix} \begin{bmatrix} \mathbf{\Delta \delta} \\ \mathbf{\Delta V} \end{bmatrix} \tag{9}$$
  From (9) and neglecting $\mathbf{J}_3$ as well as the non-diagonal elements of $\mathbf{J}_4 = \{J_{4_{ij}}\}$, the following expression is derived:
  $$\Delta Q_i \approx J_{4_{ii}} \Delta V_i = J_{4_{ii}} \left(V_i^* - V_i\right) \tag{10}$$
  where ΔQ_i is the amount of reactive compensation to be added at busbar i.
- *Stop criterion.* Computation is halted when no new nondominated solution is found to dominate an individual of the external population for a given number N_{stop} of successive generations.
- *Two External Populations.* If only one external population is used, it is possible:
  a. to save all found Pareto solutions, but this population may become too large and the evolutionary population looses genetic importance in the search process; or
  b. to loose found solutions using clustering to maintain a given number g of external solutions (original SPEA approach).

  In this proposal, two external populations are stored, one with all found nondominated solutions and another with a maximum number g of nondominated individuals, fixed by clustering, that participates in the ordinary evolutionary process. That way, the external population used in the evolutionary process does not diminish the influence of the evolutionary population and no optimal solution is lost. Note that this second external population may be stored on disk, because it does not participate in the evolutionary process.
- *Freezing.* Inspired in Simulated Annealing technique, probabilities (of mutation P_m, crossover P_c and for using the local optimization P_{lo}) change with the number of generations and fitness value, freezing at the end of the computation to improve convergence [17].

The proposed method may be summarized as follows:

1. Generate an initial population *Pop* using the heuristic method previously exposed and create two empty external nondominated sets P_{known} and SP_{known} (*stored external population*).
2. Copy nondominated members of *Pop* to P_{known} and SP_{known}.
3. Remove individuals within SP_{known}, which are covered (dominated) by any member of SP_{known}.
4. Remove solutions within P_{known}, which are covered by any member of SP_{known}.
5. If the number of externally nondominated solutions in P_{known} exceeds a given maximum g, clustering is applied in order to reduce the external population to a size g.
6. Calculate the fitness of each individual in *Pop* as well as in P_{known} using standard SPEA fitness assignment procedure.
7. Select individual from *Pop* + P_{known} (multiset union) until the mating pool is filled. In this study, roulette wheel selection is used.
8. Apply P_{lo}, P_c and P_m to determine whether and individual is locally optimized or selected for crossover and mutation, in which case, standard genetic operators are

applied.

9. Go to step 2 if stop criterion is not verified.

Figure 2 shows a flow diagram of the procedure.

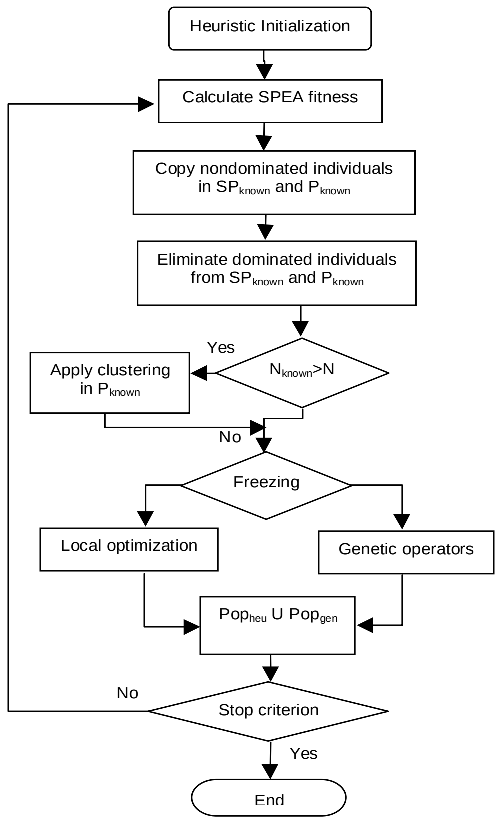

Fig. 2. Proposed Method

IV. Experimental environment

As a study case, the IEEE 118 Test Case has been selected [18] Figure 3 shows a layout of the network case. In order to stress the original system, its active and reactive loads were incremented by 40%, turning the power network in an adequate candidate for reactive power compensation.

For comparison purposes, the Pareto set generated by the proposed method has been compared to a Pareto set of compensation schemes elaborated by a team of specialized engineers using standard computational programs (*Specialist*).

For the experimental results presented in the following section, it has been assumed that $\alpha = \beta$, i.e., capacitor and reactor banks have the same cost per MVAr. At the same time, N_{stop} = 100 was experimentally chosen.

To evaluate the experimental results of the two methods, an appropriate test suite metrics is used [5], because no single metric can entirely capture total MOEA performance, effectiveness and efficiency. The test suit comprises the following metrics:

1) *Overall Nondominated Vector Generation (N)*

$$N \overset{\Delta}{=} \left| PF_{known} \right|_c \tag{11}$$

where $\left| \cdot \right|_c$ denotes cardinality.

This metric indicates the number of solutions in PF_{known}. A good PF_{known} set is expected to have a large number of nondominated individuals, in order to offer a wide variety of options for the engineer.

2) *Overall Nondominated Vector Generation Ratio* (*ONVGR*)

$$ONVGR \overset{\Delta}{=} \frac{N}{\left| PF_{true} \right|_c} \tag{12}$$

It denotes the ratio between the solution's number in PF_{known} to the number of solutions in PF_{true}. Since the objective is to obtain a PF_{known} set as similar as possible to PF_{true}, a value near to 1 is desired.

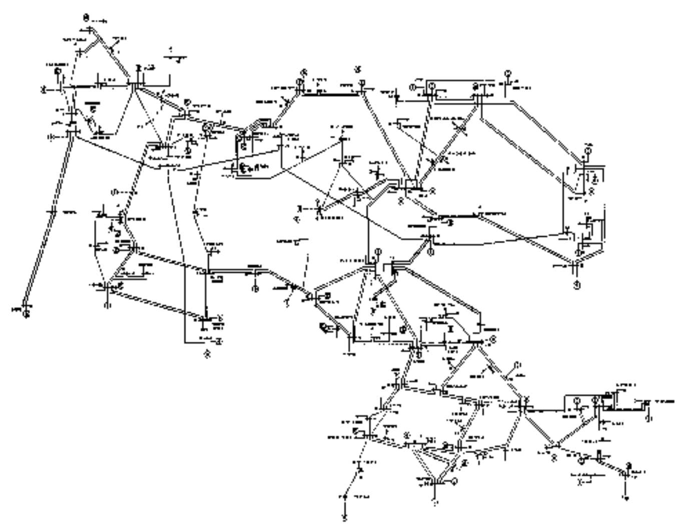

Fig. 3. IEEE 118 Busbar Test Case

3) *Error Ratio* (*ER*)

$$ER \overset{\Delta}{=} \frac{\sum_{i=1}^{N} e_i}{N} \tag{13}$$

$$e_i = \begin{cases} 0 & \text{if a vector in } PF_{known} \text{ is also in } PF_{true} \\ 1 & \text{otherwise} \end{cases}$$

This ratio reports the proportion of objective vectors in PF_{known} that are not members of PF_{true}. Therefore, an Error Ratio *ER* close to 1 indicates a poor correspondence between PF_{known} and PF_{true}, i.e., $E = 0$ is desired.

Since these metrics reflect the likeness between the true Pareto Front Optimal set PF_{true} and a computed Pareto Front set PF_{known}, a good approximation of PF_{true} is built by gathering all nondominated individuals from the sets. In other words, for the following results, PF_{true} is approximated by the best-known solutions of all the experiments.

V. EXPERIMENTAL RESULTS

Table I presents experimental results using the IEEE-118 study case, showing the figures obtained by the two methods. The proposed method has been stopped using a maximum number of generation criterion, since it continues generating new solutions reaching more than 3000 stored solutions (SP_{known}). This is an important advantage since it gives the user a wider variety of alternative solutions.

TABLE I
EXPERIMENTAL RESULTS: 100-GENERATION RUN OF THE PROPOSED METHOD

Metrics	*Specialist*	*Proposed Method*
N	170	2132
ONVGR	0.0993	1.2473
ER	0.6294	0.2265

For *N* and *ONVGR* metrics, it is clear that the proposed method has the best performance, since it generates the widest variety of solutions. Values obtained for *ER* metric show that the proposed approach generates a set of solutions that yields closer to PF_{true} than the set suggested by the specialists team.

A set of solutions generated by the proposed method is shown in Figure 4 as a graphical example. Each axis represents one objective, and each point in the surface grid is a solution of the problem.

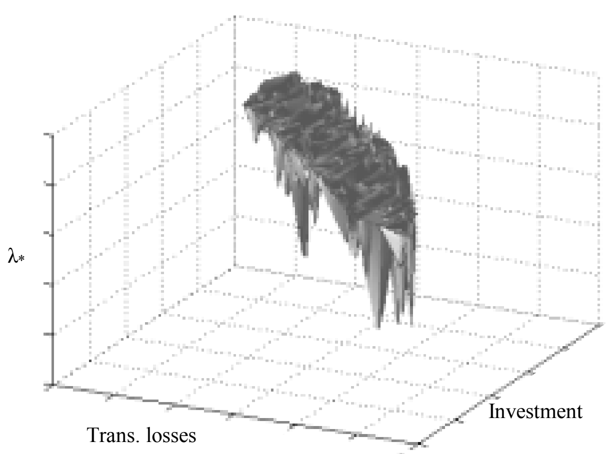

Fig. 4. Set of solutions generated by the proposed method

Figure 5 shows the solutions obtained by each method when compared using 1st and 2nd objectives. It can be seen that the proposed method is much more efficient when it is necessary to identify solutions that reduce the transmission losses, since the specialist solutions tend to higher levels of losses.

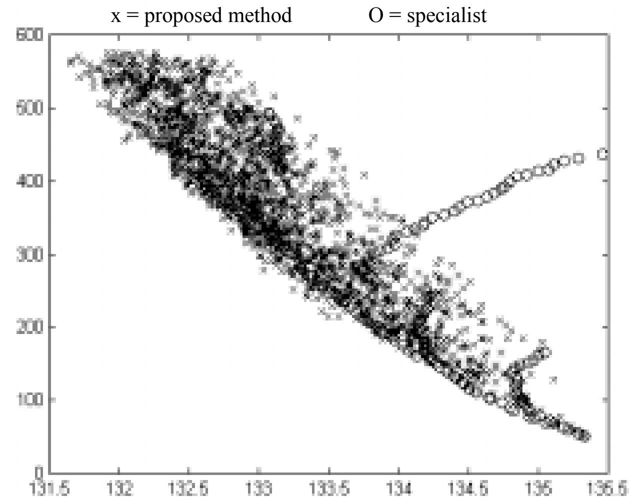

Fig. 5. Investment (MW) vs. transmission losses (MW)

Figure 6 depicts the behavior of both methods when the 2nd and 3rd objectives are evaluated. The proposed method generates solutions that are mostly concentrated in the area with low levels of transmission losses and good stability margin, while the specialist method indicates solutions that are more randomly distributed.

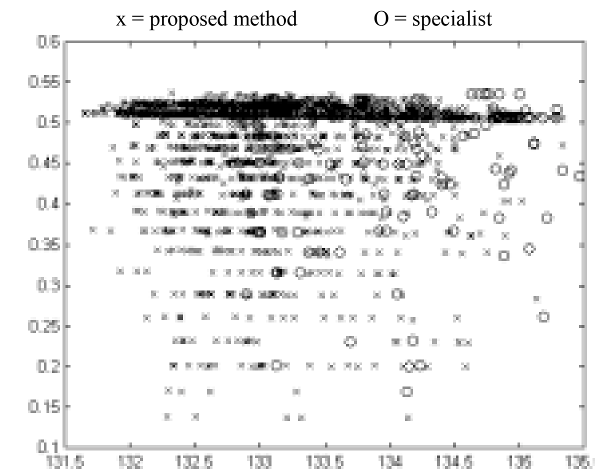

Fig. 6. Stability margin vs. transmission losses (MW)

VI. CONCLUDING REMARKS

In this paper, Reactive Compensation Problem is treated as a *Multi-objective Optimization Problem* with 3 conflicting objective functions: (i) investment in reactive compensation devices, (ii) active power losses, and (iii) voltage security.

To solve the problem, an approach based on *SPEA* is proposed. This approach introduces several proposals as: (i) heuristic initialization, (ii) a local optimization technique, (iii) a stop criterion, (iv) two external populations and (v) a freezing feature.

For comparison purposes, the solution set obtained in a single run of the proposed method is compared with the best set of solutions calculated by a team of specialists.

Experimental results using the proposed approach demonstrated several advantages when using the proposed method, such as a set of solutions closer to the *True Pareto Set* outperforming the other set in every studied figure of merits, and a wider variety of options. This last feature is of special importance, since a richer set of alternatives are offered to the network planners. In order to select sub-sets of solutions which best fit the interests of the user, an adaptive constrain philosophy is suggested. That way, the network engineer may restrict the constraints to reduce the number of solutions after having a good idea of the whole Pareto solutions, searching forward only in the redefined domain. This process may continue iteratively until a good solution with an acceptable compromise among objective functions is found.

As future work, new specialized genetic operators are being developed to locally improve reactive compensation of a given individual. At the same time, other objective functions and constraints (such as nonlinear costs of compensation devices) are going to be considered. Finally, parallel asynchronous computation using a network of computers are considered for larger networks with more objective functions.

VII. References

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VIII. Biographies

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